presented for comparison [1]. Since the characteristics are very similar for Ar, Kr and Xe, the total wall heat flux from these arcs are virtually identical. Since the electrical resistance, and hence the field intensity, for the He arc is much in excess of that for Ar, Kr and Xe, the total wall heat flux for He will exceed the values for the other gases. For each of these gases, the total wall heat flux, as well as the total wall heat transfer per unit length of tube (W/m), decreases significantly with increasing radius [5]. Although the total wall heat flux increases with pressure for Ar, Kr and Xe, it decreases with increasing pressure for He [5].

For use of the arc as a radiation source, a quantity of particular interest is the fraction of the total heat loss due to radiation. This is shown as a function of arc current in Fig. 4. The merit of the Xe arc is readily apparent. In contrast to negligible radiation from He and peak radiation efficiencies of 53 and 62 per cent for Ar and Kr, respectively, the radiation efficiency of Xe is approximately 75 per cent for the indicated operating conditions. Although there is little effect of changing tube radius, the radiation efficiency is increased significantly with increasing arc pressure. For a 1 cm dia Xe arc operating at 100 A and 10 atm, the calculations suggest a conversion efficiency of 99 per cent [5].

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EFFECT OF RADIAL VELOCITY COMPONENT ON LAMINAR FORCED CONVECTION IN ENTRANCE REGION OF A CIRCULAR TUBE

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(*Received* 1 *March* 1973 *and in revised form* 26 *June* 1973)

NOMENCLATURE

a,	tube radius;
D,	tube diameter;
I_0, I_1, I_2,	modified Bessel functions;
Nu,	Nusselt number;
Pr,	Prandtl number, ν/α;
R, X,	cylindrical coordinates;
r, x,	dimensionless cylindrical coordinates;
Re,	Reynolds number, $U_m D/\nu$;
T,	temperature;
U, V,	velocity components in X and R directions;
β,	parameter, a function of X alone;
θ,	dimensionless temperature, $(T - T_w)/(T_0 - T_w)$.

Subscripts

m,	mixed mean value or mean value;
0,	condition at tube inlet;
w,	condition at tube wall;
x,	local value.

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INTRODUCTION

DUE TO mathematical difficulties, steady laminar flow in hydrodynamic entrance region of tubes and ducts does not have any exact solution. So far, several approximations have been devised to solve the problem. One of the earliest investigations was done by Langhaar [1] in 1942. By means of a linearizing approximation, the Navier–Stokes equations were solved for the case of steady flow in the transition length of a straight tube. Han [2] and Sparrow *et al.* [3] applied this approximation to the cases of rectangular ducts and parallel-plate channels, respectively. But no detailed information was given about velocity components in the directions normal to the axial flow in the above papers [1–3].

Considering the laminar convection in combined hydrodynamic and thermal entrance region of tubes, kays [4] utilized the Langhaar's axial velocity profile [1] in the energy equation without considering the convective term in radial direction and obtained heat transfer results for three boundary conditions. He claimed that at $4\,(X/D)/(Pr\,Re) = 0{\cdot}004$ the convective term in radial direction is only 10 per cent of that in axial direction in energy equation at one point in the flow cross section, and that if the convective term in the radial direction is neglected, the local Nusselt number evaluated from calculated temperature profiles will tend to be low, and the error should be less than 5 per cent at $4(X/D)/(Pr\,Re) = 0{\cdot}004$. Experimental work was done only for constant wall temperature at $4(X/D)/(Pr\,Re) \geqslant 0{\cdot}06$ and for constant heat input at $4(X/D)/(Pr\,Re) \geqslant 0{\cdot}02$. Heaton *et al.* [5] made an analysis of the problem of laminar flow heat transfer in an annulus with simultaneously developing velocity and temperature distributions and constant wall heat flux. Kays summarized the above results and reported in [6].

The purpose of this investigation is to present the effect of radial velocity component on laminar forced convection in combined hydrodynamic and thermal entrance region of a circular tube with two boundary conditions. As a result of the radial direction convective motion, the Nusselt numbers are found to be lower than those of Kays' solution which neglected the effect of radial velocity component and this contradicts Kays' speculation [4]. The present results are also confirmed by a complete finite-difference technique using fine mesh sizes.

THEORETICAL ANALYSIS

Consider a steady laminar flow in combined hydrodynamic and thermal entrance region of a circular tube under some prescribed thermal boundary conditions. According to Langhaar's solution [1], the velocity component in the axial direction is

$$U(R,X) = U_m[I_0(\beta a) - I_0(\beta R)]/I_2(\beta a). \tag{1}$$

In this investigation the above Langhaar's profile (1) will be employed to compute the radial velocity component from the continuity equation. The solution for radial velocity component is

$$V(R,X) = -\,\nu R/(\beta a^3)\,[I_2(\beta a)\{I_0(\beta a) - I_0(\beta R)\} + I_1(\beta a)\{aI_1(\beta R)/R - I_1(\beta a)\}]/[\{I_2(\beta a)\}^2 \partial x/\partial(\beta a)]. \tag{2}$$

It is noted that the above expression of V satisfies boundary conditions at $R = 0$ and $R = a$, and the evaluation of the term $\partial x/\partial(\beta a)$ will be referred to [1]. The energy equation is solved by employing the Crank–Nicolson finite-difference method. The numbers of divisions in axial and radial directions are the same as the ones utilized in [4].

In order to confirm the accuracy of the present solution. This problem is resolved by a complete Crank–Nicolson finite-difference method for the case of $Pr = 0{\cdot}7$ and $T_w =$ constant. The mesh sizes in radial and axial directions are tabulated in Table 1, where $r = R/a$ and $x = 4X/(ReD)$.

Table 1. Mesh sizes in radial and axial directions

r \ x	0 ~ 0·001	0·001 ~ 0·01	0·01 ~ 0·1	0·1 ~ 1
1/30	0·0001/3	0·001/3	0·01/3	0·1/3
1/40	0·0001/4	0·001/4	0·01/4	0·1/4

FLOW AND HEAT TRANSFER RESULTS

Figure 1 shows variations in ratio of radial convective term and axial convective term, $(V\partial T/\partial R)/(U\partial T/\partial X)$, with dimensionless axial position, x, at different values of dimensionless radial position, R/a. In this figure, we see that the magnitude of the curve $R/a = 0{\cdot}9$ goes as high as 0·4 at $x = 0{\cdot}001$ and as x increases, the value decreases very gradually. At axial positions close to the fully developed

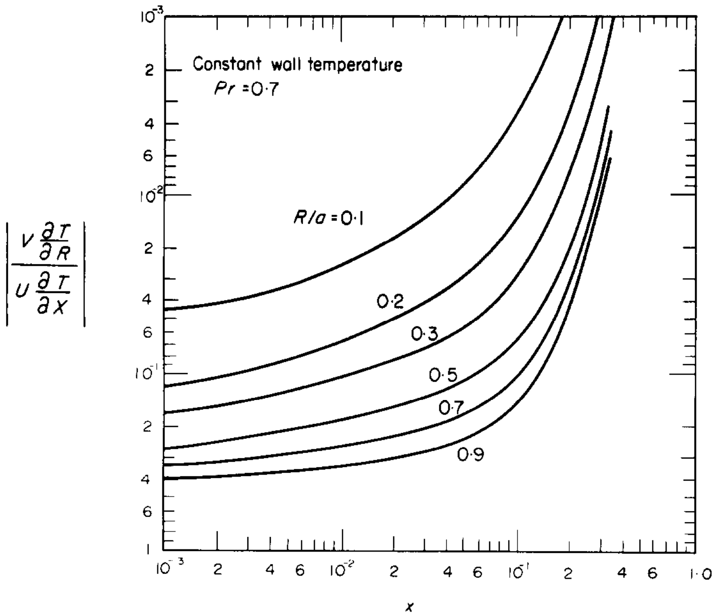

FIG. 1. $|V\partial T/\partial R)/(U\partial T/\partial X)|$ vs x.

region, say $x = 0{\cdot}1$, the ratio in some flow cross section is still greater than 0·1. We can conclude that the effect of radial direction convective motion may not be ignored in comparison with the convective motion in axial direction in most of the entrance region.

curve of local Nusselt number with $V \neq 0$ starts to deviate from the one with $V = 0$ at $x = 0{\cdot}06$. As x decreases, the difference becomes pronounced. A difference in the values of local Nusselt numbers of 40 per cent based on the present result is found at $x = 0{\cdot}001$. The same situation is also

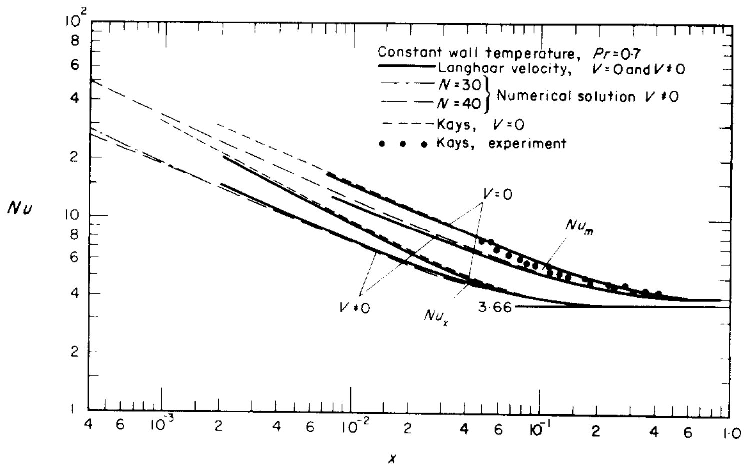

FIG. 2. Nusselt numbers for the case of $Pr = 0{\cdot}7$ and constant wall temperature.

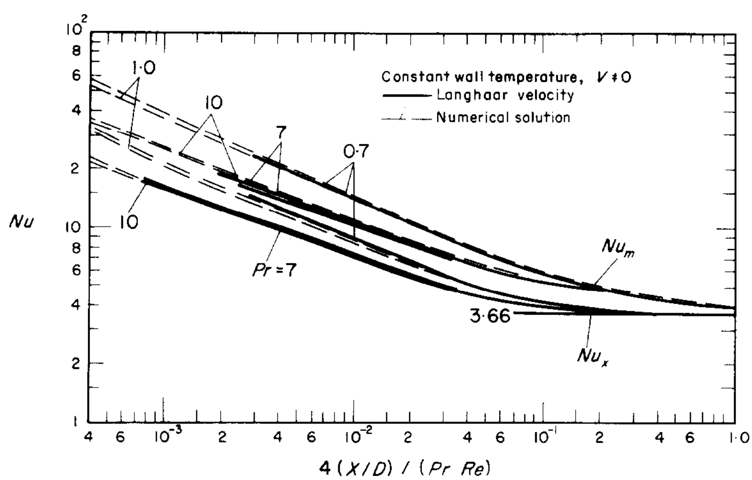

FIG. 3. Nusselt numbers for the case of constant wall temperature.

According to the definitions in reference [4], the local Nusselt number, Nu_x, and mean Nusselt number, Nu_m, for the case of $Pr = 0{\cdot}7$ and constant wall temperature are plotted in Fig. 2. In order to confirm the present result, solution with $V = 0$ was calculated and these curves check well with the ones obtained by Kays [4] for both local and mean Nusselt numbers. The accuracy of the solution with $V \neq 0$ is also assured by a complete numerical solution. When the effect of radial velocity component is considered, the observed in the values of mean Nusselt numbers. Ranging from $x = 0{\cdot}05$ to $x = 0{\cdot}4$, there are individual points showing the experimental result obtained by Kays [4]. The experimental data with 5 per cent uncertainty, in general, lie between the curves considering or neglecting the effect of radial velocity component.

Figure 3 shows values of Nusselt number for the case of constant wall temperature with Prandtl number as a parameter. We present here the solution obtained using Langhaar

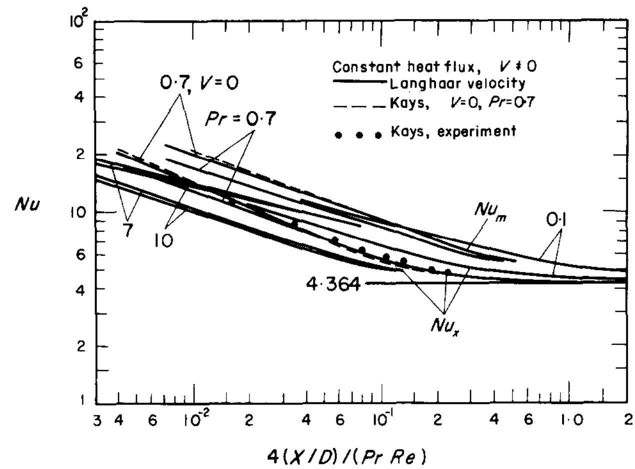

FIG. 4. Nusselt numbers for the case of constant heat flux.

velocity profile (1) and equation (2) and the solution obtained by complete numerical method. Good agreement between these two solutions is found for the values of local Nusselt numbers. But due to the integration of local Nusselt number in equation (4), a slightly larger error is observed for the curves of mean Nusselt number.

The Nusselt numbers for the case of constant heat flux and $Pr = 0{\cdot}1, 0{\cdot}7, 7$ and 10 are presented in Fig. 4. Experimental data, discussed by Klein and Tribus [7], are also reproduced from Kays [4]. The range of these data is from $x = 0{\cdot}015$ to $x = 0{\cdot}2$ in which the difference between the solutions considering and neglecting the effect of radial velocity component is small.

CONCLUSIONS

1. Although the magnitude of radial velocity component in comparing with the one in axial direction is small, the effect of radial convective motion as shown in Fig. 1 cannot be ignored in combined hydrodynamic and thermal entrance flow region. With the effect of radial velocity component, the value of heat transfer coefficient is lower than the one obtained neglecting the radial velocity component. This prediction contradicts with the speculation made by Kays [4].
2. It is believed that similar effect of radial velocity component will be found in parallel plate channel, rectangular channel or different geometrical cross sections.

Acknowledgement—The authors would like to thank Father E. Matis for reading of this manuscript and thank the computing center at National Tsing Hua University for providing computing time.

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